UDC 656.02

MODEL AND METHOD FOR CALCULATING MATRICES OF CORRESPONDENCES TAKING INTO ACCOUNT TIME LIMITS OF TRAVEL BENEFITS

Slavych V. P., *Ph.D., Associate Professor, Associate Professor of the Department of Transport Systems and Technical Service of Kherson National Technical University, Kherson, Ukraine, e-mail: vslavich@ukr.net, ORCID: 0000-0001-7882-4198.*

The article proposes an improved model for calculating matrices of passenger correspondences, which takes into account time limits of travel benefits. The relevance of the study is due to the need to adapt transport planning to social changes and reduce passenger traffic in conditions of limited funding. Traditional approaches do not take into account the impact of changes in the time of benefits, which leads to distortion of modeling results and inefficient use of resources. The proposed model takes into account the temporal availability of benefits and allows calculating the attraction coefficients between transport areas, taking into account the periods of paid and free travel. Special attention is paid to the analysis of the dynamics of trips of persons of preferential categories at different times of the day, which allows correctly considering peak and interpeak loads. The introduced parameters allow calculating the share of time allocated to each type of travel and combining the corresponding attraction coefficients. Thus, the model reflects the change in the mobility of preferential categories of passengers depending on the time of day, which is especially important for cities with a high share of such passengers. The developed approach also allows assessing potential changes in passenger flows when adjusting the time limits of the benefits, which can be useful for transport management bodies. The paper considers various options for the mutual arrangement of the time intervals of the benefits and the calculation of the corresponding weighting coefficients. The results obtained allow the formation of more accurate and flexible correspondence matrices that take into account social policy in the field of transportation, demand reduction and uneven load on the transport network. The proposed approach is universal in nature and can be adapted to different conditions of public transport operation, contributing to more effective planning of routes and schedules. In addition, the model can be integrated into existing transport modeling information systems for automated consideration of preferential factors.

Key words: *passenger transportation; correspondence matrix; route transportation; preferential travel; transport modeling; passenger flow; time constraints; attraction coefficient; public transport; trip dynamics; transportation planning.*

DOI: 10.33815/2313-4763.2025.1.30.187-194

Introduction. The modern transport system faces challenges related not only to the technical aspects of transportation, but also to changes in social policy, in particular in the field of preferential travel. In conditions of limited funding and the need for rational use of resources, the calculation of correspondence matrices that take into account the time limits of the benefits is becoming increasingly relevant. Traditional transport planning models are not always able to accurately reflect the impact of such restrictions on the spatio-temporal structure of passenger flows, which leads to distortions in forecasting and inefficient transport management.

The need to improve the methods of calculating correspondence matrices is due to the need to increase social justice, economic efficiency and optimize transport costs. This issue is of particular importance in cities with a high proportion of passengers using benefits that are limited in time (for example, only during off-peak hours). In such cases, the structure of demand shifts, which must be taken into account in analytical models to ensure adequate planning of routes, traffic schedules and vehicle loads.

Thus, the development of new models capable of integrating time constraints on benefits into the process of constructing correspondence matrices is not only relevant but also critically necessary for creating an efficient, flexible, and socially oriented transportation system.

Analysis of recent achievements and publications. Research into methods for generating and analyzing passenger correspondence matrices is a key area of research in transport modeling, especially when considering time constraints associated with travel benefits. Different approaches to calculating such matrices allow us to choose the most appropriate models for specific urban mobility conditions.

In the works [1, 3, 4] synthetic and stochastic methods of constructing correspondence matrices that take into account the structural features of passenger flows are considered. In [3] the formulation of the problem of random filling of the matrix is proposed, considering the uncertainty in the initial data, which is especially relevant in the case of preferential transportation, when accurate observation of the passenger is difficult. In [4] the accuracy of models for small cities is estimated, where the effects of the influence of privileges can be more pronounced due to the smaller number of alternative routes.

Interval approaches to modeling that consider ranges of possible values instead of fixed ones are discussed in [2]. This approach allows for better consideration of temporal variability in demand, particularly in the case of limited-time benefits.

The works [6, 7, 9] are devoted to modeling the dynamics of transport and pedestrian flows, as well as determining the passenger waiting time, which is of critical importance for analyzing the behavior of preferential categories of passengers who may change routes in response to changes in service conditions. In [6], a model for optimizing the stay of transport at stops is considered, which allows considering the time component when forming correspondences.

The issue of accounting for preferential passenger categories in urban conditions is partially considered in [8], which examines the impact of reducing the number of carriers on passenger correspondence. This allows us to draw conclusions about the flexibility of passenger demand and changes in the structure of trips due to restrictions on transport supply.

The application of intelligent systems for passenger flow forecasting is presented in [11], where electronic ticket data is used to model passenger flow during special events. Such approaches can be adapted to model the behavior of preferential passengers depending on the time of day or calendar day.

Special attention is paid to modern works that combine methods of network analysis and transport geography [10, 14, 15]. It is shown how weighting factors in complex networks allow considering real passenger traffic, which can also be used to analyze the impact of time restrictions on the activity of preferential categories of transport users.

In [12], a method for integrating ticket sales and passenger registration data to construct dynamic OD matrices was developed. This approach is extremely promising in the context of studying preferential travel, when taking into account the time of day is of fundamental importance. And in [13], optimization of transport frequency is considered, considering new restrictions that arose during the pandemic; similar methods can be applied to adapting transport to the time frame of the benefits.

Thus, the analyzed sources form a comprehensive picture of modern approaches to modeling passenger correspondence, especially in conditions of time and social constraints. The proposed study is a logical continuation and expansion of the above approaches, aimed at taking into account the specifics of the action of travel benefits in the temporal aspect.

Goal and problem statement. The aim of the work is to develop a model for calculating correspondence matrices that takes into account restrictions in the form of time changes in the validity of travel privileges. This approach involves the development of methods for adapting to reduced demand, optimizing the frequency of flights, and maintaining the efficiency of transfers between different modes of transport.

Presentation of the main research material. Consider the transport network of a hypothetical city, which consists of separate transport districts or zones, between which there is intensive daily movement of passengers. This movement is carried out using public road transport, which provides a connection between residential, business, educational and other functional zones of the city. The transport network is considered as a system that includes nodes (transport districts) and connections between them (public transport routes), along which passenger flows move.

To simplify the analysis and focus on key aspects related to the impact of restrictions on benefits, it is advisable to introduce a generalization: consider public transport as a homogeneous system, represented by one type of vehicle – for example, buses. This allows us to abstract from technical and technological differences between modes of transport and focus on the basic patterns of passenger flow formation and their change due to the action of social restrictions. This approach does not reduce the generality of the model, but on the contrary – it allows us to identify universal dependencies that can be further adapted to broader conditions.

Let us consider the construction of a generalized model that allows calculating the matrix of passenger correspondences taking into account the time limits for the validity of travel privileges.

The model parameters are presented in Table 1.

Table 1 – Model parameters

No.	***Designation parameter***	***Parameter name***	***Unit measurement parameter***
Model input parameters			
1	$n_л$	Number of passengers in preferential categories traveling within the transport network	Persons
2	p	Statistical constant reflecting the ability of persons from privileged categories to travel for a fee in the transport network	–
3	t_0	Time of entry of vehicles onto the transport network route	Time units
4	$t_к$	Time of return of vehicles to the depot (end of route)	Time units
5	$t_{л1}$	Start time of free travel for preferential categories of passengers traveling in a given transport network	Time units
6	$t_{л2}$	Expiration time of free travel for preferential categories of passengers traveling in a given transport network	Time units
7	$[t_1;t_2]$	The time period over which the correspondence matrix is calculated	Time units
8	t	Current time	Time units
Intermediate model parameters			
9	c_{ij1}	Gravity coefficient taking into account passengers of preferential categories during paid travel	–
10	c_{ij2}	Attraction coefficient taking into account passengers of preferential categories during free travel	–
11	Θ_1	Percentage coefficient of finding the time period on which the calculation is made in paid parts of the day	–
12	Θ_2	Percentage of finding the time period on which the calculation is made in free parts of the day	–

In modern conditions, society is increasingly faced with the influence of various external factors that significantly change the usual rhythm of life. One of such factors is the epidemiological situation, which can lead to restrictions on the activities of certain areas of social functioning, in particular, transport. In such conditions, optimization of transport connections and management of passenger flows becomes of particular importance. One of the most relevant restrictions that affect the formation of transport demand is the limitation of the duration of travel privileges for certain categories of passengers.

Such restrictions lead to a change in the nature of movement of preferential categories of citizens, causing uneven load on the transport system at different times of the day. In this regard, traditional approaches to the formation of correspondence matrices – one of the main tools of transport modeling – need to be adapted and improved. In particular, it is advisable to introduce a variable attraction coefficient, which depends on the presence or absence of privileges in a specific period of time. This will allow for a more accurate account of the change in demand due to time restrictions on free travel.

Let t_0 and t_k denote the start and end times of the public transport route, respectively. In this case, the time interval from t_{l1} to t_{l2} denotes the range of validity of the preferential travel – that is, the period during which passengers of preferential categories have the right to free travel. It is obvious that if the moment of the trip does not fall into this interval, the probability of the trip for the beneficiary decreases, which must be reflected in the model by means of an appropriate adjustment of the attraction coefficients in the correspondence matrix.

Let's create a system:

$$\begin{cases} c_{ij1} = \dfrac{n_л}{D_j} \cdot p + \dfrac{D_j - n_л}{D_j} \cdot (t_п^{-1} \cdot s^{-1}), \text{ якщо } \left[\begin{array}{l} t1; t2 \le t_{л1} \\ \\ t \ge t_{л2} \end{array}\right. ; & (1) \\ \\ c_{ij2} = \dfrac{n_л}{D_j} \cdot (t_п^{-1}) + \dfrac{D_j - n_л}{D_j} \cdot (t_п^{-1} \cdot s^{-1}), \text{ якщо } \begin{cases} t_{л1} \le t1 \\ \\ t2 \le t_{л2} \end{cases}. & (2) \end{cases}$$

That is, during the period of the benefits, when travel for certain categories of passengers is free, the attraction coefficient c_{ij} between districts i and j for such passengers does not depend on the cost of the trip. During this period, only the spatial and temporal characteristics of the route remain decisive factors – distance, duration of the trip, frequency of transport, etc. At the same time, when a trip for preferential categories becomes paid, that is, it goes beyond the allowed free period – there is a decrease in the mobility of this group of passengers. In such cases, the attraction coefficient is no longer neutral, but has a conditionally constant weight, which can be denoted as a statistical constant p, which characterizes the willingness of passengers of preferential categories to make paid trips.

In other words, during the time of the benefit (for example, during off-peak hours of the day), the demand for trips from benefit holders remains high, while at other times of the day – in particular during rush hours or late at night – this demand decreases significantly. The reason for this is not only financial, but also a change in motives for travel – many benefit holders plan their trips taking into account the time of free travel.

In this regard, for the correct calculation of the correspondence matrix with the participation of passengers of preferential categories, it is necessary to take into account the time limits of the validity or absence of privileges. That is, it is necessary to know exactly for what period of time the matrix is calculated, as well as what proportion of this interval falls on periods of free or paid travel. Otherwise, the model will lose its adequacy and will not be able to correctly reflect the real behavior of passengers, which will lead to data distortion and errors in transport planning.

That is why the following variable parameters are introduced:

Θ_1 – shows the part of the time period that is in the paid part of the day,

Θ_2 – shows the part of the time period that lies in the free part of the day.

Let us consider all possible options for calculating the variables Θ_1 and Θ_2.

Under the condition t_1, t_2, $t_{л1}$, $t_{л2} \in [t_0; t_к]$ then we will compose the systems.

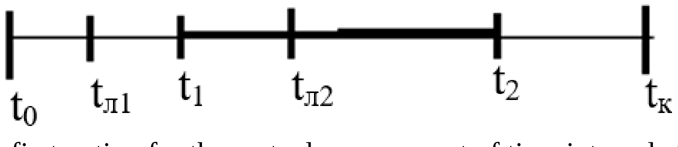

Figure 1 – The first option for the mutual arrangement of time intervals travel benefits

When performing conditions:

$$\begin{cases} t_2 > t_{л2} \\ t_1 > t_{л1} \end{cases},$$

we get:

$$\Theta_1 = \frac{t_2 - t_{л2}}{t_2 - t_1},$$

$$\Theta_2 = 1 - \Theta_1.$$

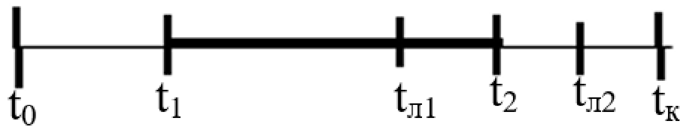

Figure 2 – Second option for the mutual arrangement of time intervals travel benefits

When performing conditions (Fig. 2):

$$\begin{cases} t_2 < t_{л2} \\ t_1 < t_{л1} \end{cases},$$

we get:

$$\Theta_1 = \frac{t_{л1} - t_1}{t_2 - t_1},$$

$$\Theta_2 = 1 - \Theta_1.$$

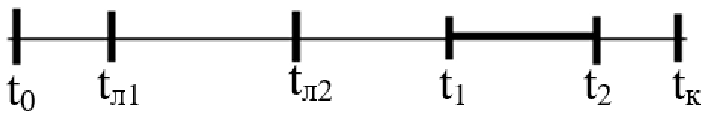

Figure 3 – Third option for the mutual arrangement of time intervals travel benefits

When performing conditions (Fig. 3):

$$\begin{cases} t_2 > t_{л2} \\ t_1 > t_{л2} \end{cases},$$

we get:

$$\Theta_1 = 1,$$

$$\Theta_2 = 0.$$

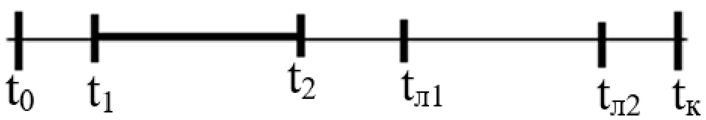

Figure 4 – Fourth option for the mutual arrangement of time intervals travel benefits

When performing conditions (Fig. 4):

$$\begin{cases} t_2 < t_{л1} \\ t_1 < t_{л1} \end{cases},$$

we get:

$$\Theta_1 = 1,$$

$$\Theta_2 = 0.$$

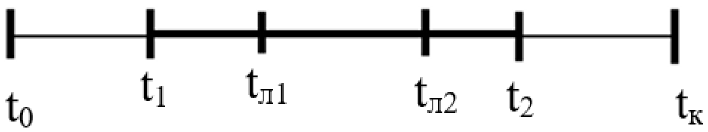

Figure 5 – Fifth option for the mutual arrangement of time intervals travel benefits

When performing conditions (Fig. 5):

$$\begin{cases} t_2 > t_{л2} \\ t_1 < t_{л1} \end{cases},$$

we get:

$$\Theta_1 = \frac{t_{л1} - t_1}{t_2 - t_1} + \frac{t_2 - t_{л2}}{t_2 - t_1},$$

$$\Theta_2 = 1 - \Theta_1.$$

Figure 6 – The sixth option for the mutual arrangement of time intervals travel benefits

When performing conditions (Fig. 6):

$$\begin{cases} t_2 < t_{л2} \\ t_1 > t_{л1} \end{cases},$$

we get:

$$\Theta_1 = 0,$$

$$\Theta_2 = 1.$$

Then the final formula for calculating the gravity coefficient is in case of limiting the validity period of travel benefits will be found using the expression:

$$c_{ij} = \Theta_1 \cdot c_{ij1} + \Theta_2 \cdot c_{ij2}, \tag{4}$$

where: c_{ij1} is calculated by formula (1), $c_{ij2}$by formula (2) .

Then, considering

$$c_{ij} = \Theta_1 \cdot \left(\frac{n_л}{D_j} \cdot p + \frac{D_j - n_л}{D_j} \cdot (t_п^{-1} \cdot s^{-1}) \right) +$$
$$+\Theta_2 \cdot \left(\frac{n_л}{D_j} \cdot (t_п^{-1}) + \frac{D_j - n_л}{D_j} \cdot (t_п^{-1} \cdot s^{-1}) \right). \tag{5}$$

Conclusions.

1. Thus, the paper proposes a model for calculating passenger correspondence matrices that considers time restrictions on travel privileges. Such solutions will provide more accurate forecasting of passenger flows and more efficient use of limited transport network resources, considering the decrease in demand for certain routes and the limited number of available carriers. The model allows you to optimize the frequency of flights by changing the intervals between trips, as well as redistribute passenger flows between different routes, which allows you to maintain a high level of service under conditions of reduced transportation.

2. Taking into account the time limits of travel benefits in the process of constructing correspondence matrices is an important stage in improving transport modeling. This allows for a more accurate reflection of the real conditions of passenger flow, taking into account social factors

and changes in the demand structure. The development and implementation of new models taking into account such restrictions contributes to increasing the accuracy of forecasting, optimizing transport planning, as well as a more rational allocation of resources. This approach ensures not only the economic efficiency of transportation, but also contributes to the formation of a more fair and adaptive transport policy focused on the needs of different categories of the population.

Prospects for further research. Further research consists in expanding the proposed model by including additional socio-demographic parameters of passengers, as well as taking into account seasonal and daily fluctuations in demand. It is advisable to investigate the possibility of integrating the model with real-time operational monitoring systems of passenger flows for adaptive management of the route network. Special attention should be paid to the development of algorithms for automatic adjustment of travel intervals based on predicted changes in the structure of preferential transportation. Further research may focus on a comparative analysis of the effectiveness of the model in conditions of different types of urban transport systems.

REFERENCES

1. Bilous, A. B., Demchuk, I. A. (2014). Analiz metodiv ta modelei rozrakhunku obsiahu pasazhyrskykh korespondentsii. *Skhidno-Yevropeiskyi zhurnal peredovykh tekhnolohii*. № 3(3). P. 53–57.

2. Horbachov, P. F., Liubyi, Ye. V., Kovtsur, K. H., Tsyn, S. (2024). Shchodo pytannia modeliuvannia elementiv matryts korespondentsii v ramkakh intervalnoi kontseptsii formuvannia modelei transportnoho popytu. Napriamky rozvytku tekhnolohichnykh system i lohistyky v APV. Materialy V-yi Mizhnarodnoi naukovo-praktychnoi internet-konferentsii (23 travnia 2024). Kharkiv: DBTU. P. 48–49.

3. Liubyi, Ye., Kovtsur, K., Tsyn, S. (2024). Postanovka zadachi vypadkovoho zapovnennia matrytsi pasazhyrskykh korespondentsii. *Suchasni tekhnolohii v mashynobuduvanni ta transporti*. № 2(23). P. 152–158.

4. Liubyi, Ye. V., Kolii, O. S. (2019). Otsinka tochnosti syntetychnykh modelei rozrakhunku pasazhyrskykh korespondentsii na prykladi malykh mist. *Suchasni tekhnolohii v mashynobuduvanni ta transporti*. № 1. P. 98–106.

5. Ponkratov, D. P., Faletska, H. I. (2015). Vybir pasazhyramy shliakhiv peresuvannia v mistakh : monohrafiia. Kharkiv : KhNUMH im. O. M. Beketova. 164 p.

6. Slavych, V. P., Hirik, D. O. (2019). Model optymizatsii chasu perebuvannia hromadskoho transportu na zupynkakh. *Visnyk KhNTU*. № 2(69). P. 187–191.

7. Slavych, V. P., Livandovskyi, V. S. (2021). Model systemy upravlinnia pishokhidnym potokom mista. *Visnyk KhNTU*. № 2(77). P. 47–51.

8. Slavych, V. P., Marchuk, N. V. (2023). Model vyznachennia pasazhyrskykh korespondentsii m. Kherson v umovakh zmenshennia kilkosti pereviznykiv. *Synerhiia nauky i biznesu u povoiennomu vidnovlenni Khersonshchyny* : materialy Mizhnarodnoi nauk.-prakt. konf. Odesa, P. 316–318.

9. Chyzhyk, V. M. (2019). Rozrobka analitychnykh modelei vyznachennia chasu ochikuvannia pasazhyramy marshrutnoho transportu v mistakh : avtoref. dys. ... kand. tekhn. nauk : 05.22.01. Kharkiv. 20 p.

10. Ding Luo, Oded Cats, Hans van Lint, Graham Currie. (2019). Integrating network science and public transport accessibility analysis for comparative assessment. *Journal of Transport Geography*. Vol. 80. 102505.

11. Chen, E., Ye, Z., Wang, C., Xu, M. (2020). Subway passenger flow prediction for special events using smart card data. *IEEE Transactions on Intelligent Transportation Systems*. Vol. 21, No 3. P. 1109–1120.

12. Greta Galliani, Piercesare Secchi, Francesca Ieva (2024). Estimation of dynamic Origin–Destination matrices in a railway transportation network integrating ticket sales and passenger count data. *Transportation Research Part A: Policy and Practice*. Vol. 190. 104246.

13. Konstantinos Gkiotsalitis, Oded Cats (2022). Optimal frequency setting of metro services in the age of COVID-19 distancing measures. *Transportmetrica A: Transport Science*. Vol. 18, No 3. P. 807–827.

14. Wang, L.-N., Wang, K., Shen, J.-L. (2020). Weighted complex networks in urban public transportation: Modeling and testing. *Physica A: Stat. Mech*. Vol. 545. 123498.

15. Tina Šfiligoj, Aljoša Peperko, Patricija Bajec, Oded Cats (2025). Node importance corresponds to passenger demand in public transport networks. *Physica A: Statistical Mechanics and its Applications.* Vol. 659. 130354.

Славич В. П. МОДЕЛЬ ТА МЕТОД РОЗРАХУНКУ МАТРИЦЬ КОРЕСПОНДЕНЦІЙ З УРАХУВАННЯМ ЧАСОВИХ ОБМЕЖЕНЬ ДІЇ ПРОЇЗНИХ ПІЛЬГ

У роботі запропоновано вдосконалену модель розрахунку матриць пасажирських кореспонденцій, що враховує часові обмеження дії проїзних пільг. Актуальність дослідження зумовлена потребою адаптації транспортного планування до соціальних змін та зменшення пасажиропотоку в умовах обмеженого фінансування. Традиційні підходи не враховують вплив змін у часі дії пільг, що призводить до викривлення результатів моделювання та неефективного використання ресурсів. Запропонована модель враховує часову доступність пільг і дозволяє розраховувати коефіцієнти тяжіння між транспортними районами з урахуванням періодів платного та безкоштовного проїзду. Особливу увагу приділено аналізу динаміки поїздок осіб пільгових категорій у різні години доби, що дозволяє коректно врахувати пікові та міжпікові навантаження. Введені параметри дозволяють розраховувати частку часу, що припадає на кожен тип проїзду, та комбінувати відповідні коефіцієнти тяжіння. Таким чином, модель відображає зміну мобільності пільгових категорій пасажирів залежно від часу доби, що особливо важливо для міст із високою часткою таких пасажирів. Розроблений підхід також дає змогу оцінити потенційні зміни пасажиропотоків при коригуванні часових меж дії пільг, що може бути корисним для органів управління транспорту. У роботі розглянуто різні варіанти взаємного розташування інтервалів часу дії пільг та розрахунку відповідних вагових коефіцієнтів. Отримані результати дозволяють формувати більш точні та гнучкі матриці кореспонденцій, що враховують соціальну політику у сфері перевезень, зменшення попиту та нерівномірність навантаження на транспортну мережу. Запропонована модель може бути використана для оптимізації розкладів руху автобусів у години з підвищеним попитом серед пільгових категорій (наприклад, ранкові години для пенсіонерів), для прогнозування змін навантаження на маршрути у разі корекції політики пільгового проїзду (наприклад, скасування пільг у пікові години або введення обмежень у вихідні дні), а також для аналізу ефективності використання муніципальних субсидій на перевезення у різний час доби та дні тижня. Модель також дозволяє обґрунтувати необхідність запуску додаткових рейсів або змінити маршрути з урахуванням змін у поведінці пільгових пасажирів, що сприятиме більш збалансованому навантаженню на транспортну мережу. Вона може бути корисною при формуванні соціально орієнтованої тарифної політики, яка враховує час доби та категорії пасажирів, забезпечуючи більш раціональне використання ресурсів. Крім того, модель може бути інтегрована в наявні інформаційні системи транспортного моделювання для автоматизованого врахування пільгових факторів.

Ключові слова: *пасажирські перевезення; матриця кореспонденцій; маршрутні перевезення; пільговий проїзд; транспортне моделювання; пасажиропотік; часові обмеження; коефіцієнт тяжіння; громадський транспорт; динаміка поїздок; планування перевезень.*

Статтю прийнято до редакції 19.05.2025